附件三

屏東縣第 60 屆國中小學科學展覽會
作品說明書

科　　別：數學

組　　別：國小組

作品名稱：Crazy knights

關 鍵 詞：騎士巡邏、連通圖、______（最多三個）

編號：A1020

製作說明：

1.說明書封面僅寫科別、組別、作品名稱及關鍵詞。

2.編號：由承辦學校統一編列。

3.封面編排由參展作者自行設計。

摘要

我們觀察 Crazy Knights 騎士交換規律並將其擴充得到新的棋盤遊戲，從中得到七組十連塊合計 76 個瘋狂騎士棋盤格，可完成至少一組黑白騎士交換，至多可完成四組黑白騎士交換，並得到最少移動步數。

壹、 研究動機

這是一個源自於西洋棋衍生的另類棋盤遊戲，我們觀察規律後將其擴充得到新的棋盤格。從中找出最多騎士的可能。

貳、 研究目的

一、解出 Crazy Knights 遊戲棋盤騎士交換條件。

二、研究不同連方塊引導出的騎士路徑連通特性。

三、找出可交換最多騎士的可能性。

參、 研究設備與器材

一、兩種顏色連方塊，用來組合各種連塊組合可能。

二、紀錄資料之 A4 紙張、筆記與電腦。

肆、 研究歷程

騎士路徑可以延伸之可能性，我們連續分析七連塊到八連塊，從中分析路徑，繪製節點圖，進一步從連通圖發展找出符合條件的 Crazy Knights 棋盤，並進一步找出放置最多騎士的可能性。

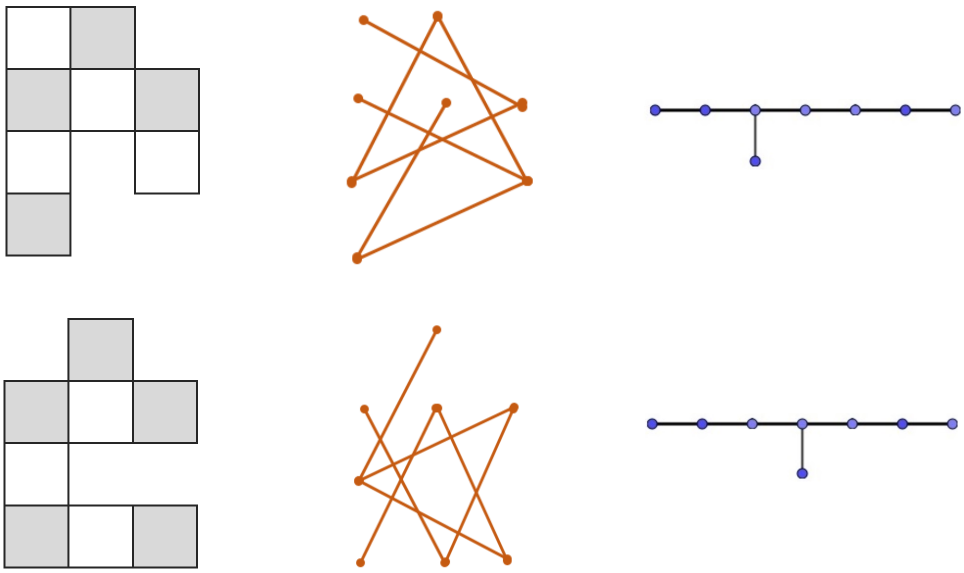
一、簡單圖

一般的圖(graph)屬性為有限個點，任兩點之間最多只有一條邊相連，有序對 G=(V,E)，V 是非空有限集合，叫做「點」(vertex)，本研究指「點集合」，E 叫做「邊」(edge)，在此為「邊集合」。圖的次數指頂點的個數，圖的尺寸指「邊集」(edge set)。

二、騎士路徑

連方塊引導出來的騎士路徑有很多種，但前提是要形成騎士巡邏才能進一步分析騎士交換的可能性。我們分析七連塊，發現七連塊只有三種連通圖，兩種為 a=b 同構騎士路徑圖，兩等價棋盤無法完成騎士交換，只有一個是兩異構圖 c, { a,b }，路徑不同，可以形成 Crazy Knights 遊戲。

三、以前導之八連塊所析出之連通圖與節點圖，異構圖特性為分叉停駐點，可進一步發展為瘋狂騎士棋盤



伍、 研究結果

我們猜想瘋狂騎士棋盤格可能不只一種，並從八連塊篩選出連通圖，思考原始路徑形成騎士巡邏的可能性。我們聚焦在有條件的重組，也就是前提必須先能形成騎士巡邏再進行重組，從篩選出來的節點圖以原始八連塊加兩虛格重組為新的棋盤遊戲。

一、重組後的十連塊

我們從八連塊找到可用的 13 個連通圖，刪除之後剩下 7 個騎士巡邏，以順時針方向加兩虛格(黃色標示)後刪除重複，得到七組十連塊。

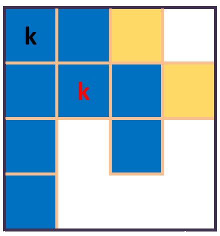
1. 中空圖形：騎士路徑可透過圖中空格飛躍到實格，但不能在中空格落點停駐，因此不算十一或十二連塊。
2. 設計出 76 個棋盤格，其中中空棋盤有 22 個。

表 1　十連塊瘋狂騎士棋盤格種類及數量

編號	A	B	C	D	E	F	G	合計
數量	11	14	7	11	14	10	9	76
中空	2	5	5	3	5	0	2	22

全新十連塊皆是連通圖。兩對立騎士呈現節點圖 ABCDEFG 七組節點圖，為了便於比較，以紅 k 表示白騎士，黑 k 表示黑騎士。

A 組　十連塊和節點圖



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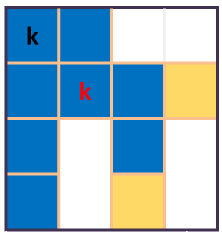

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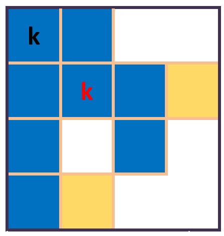

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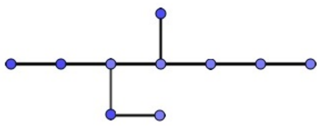

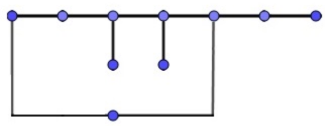

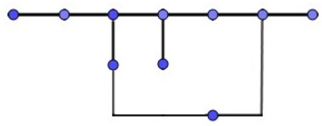

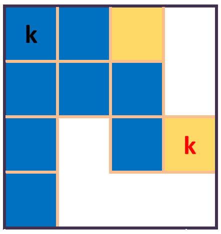

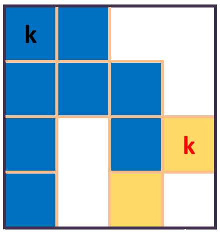

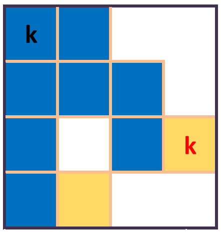

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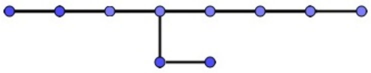

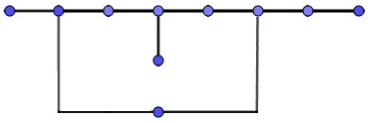

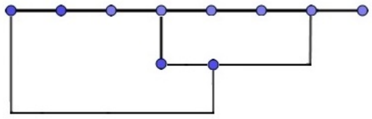

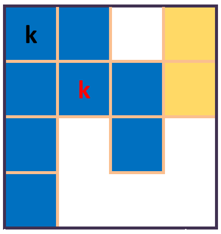

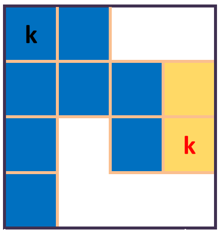

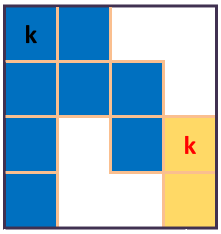

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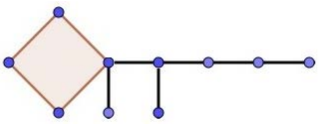

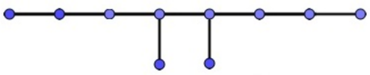

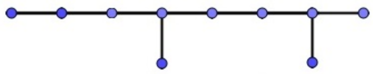

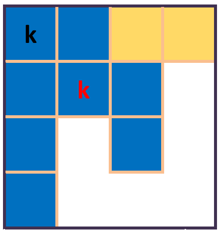

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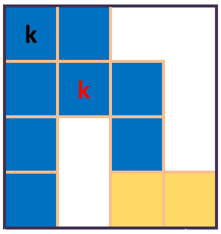

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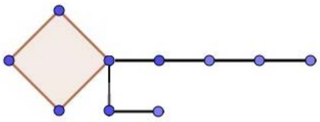
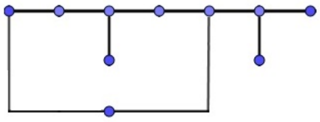
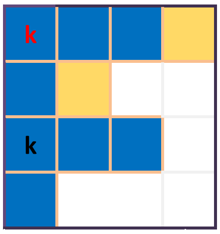




B 組 十連塊和節點圖



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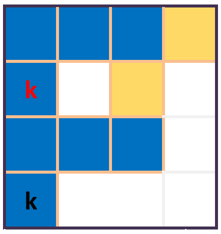

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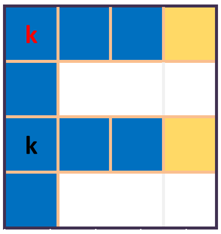

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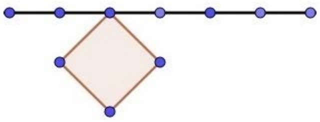

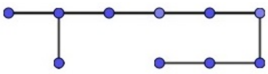

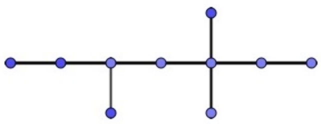

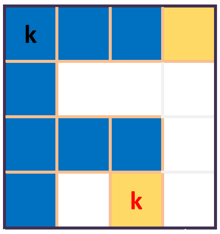

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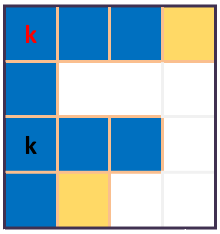

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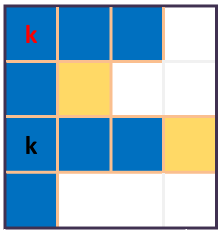

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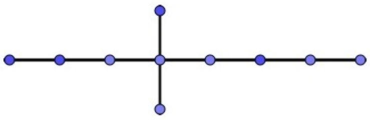

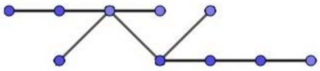

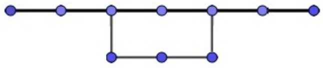

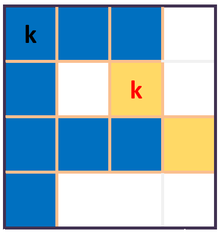

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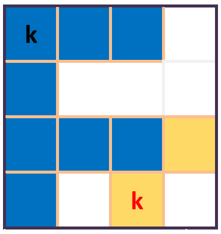

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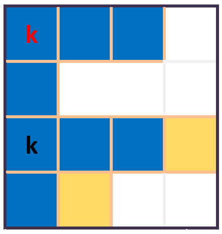

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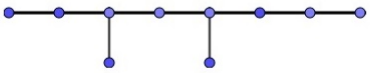

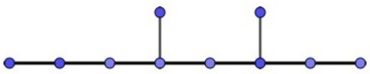

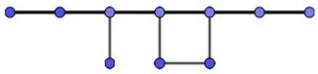

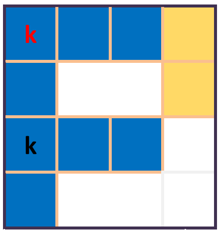

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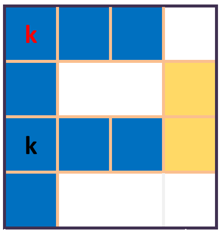

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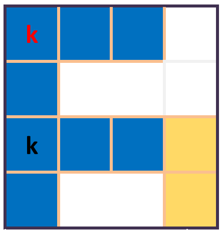

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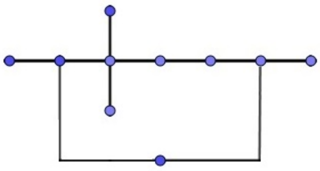

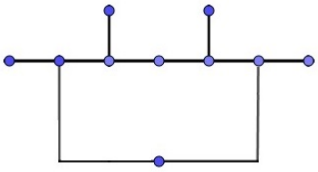

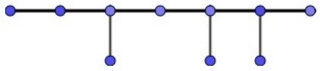

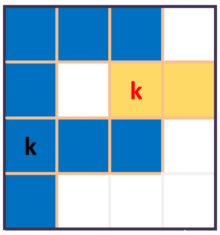

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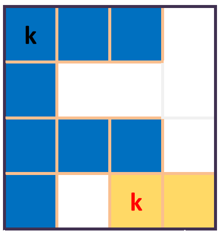

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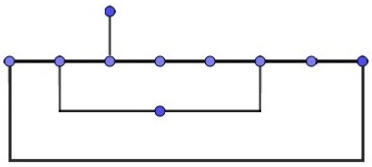
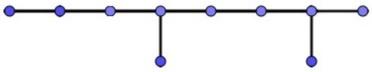
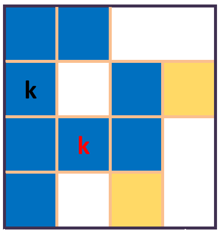




C 組 十連塊和節點圖



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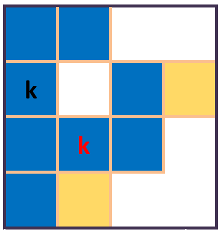

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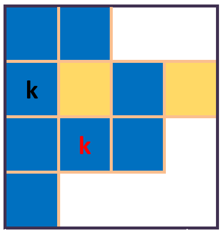

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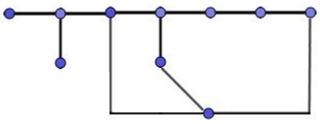

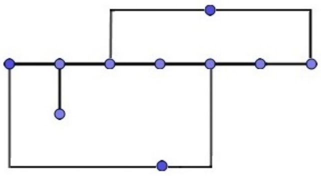

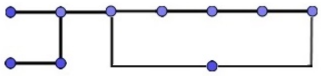

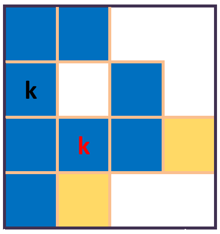

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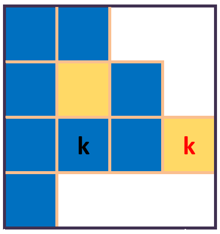

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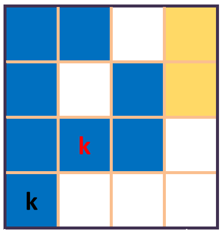

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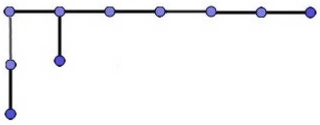

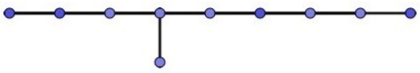

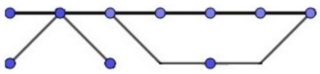

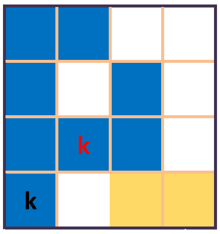

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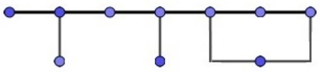
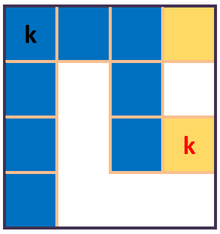


D 組 十連塊和節點圖



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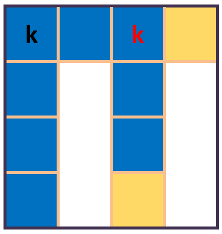

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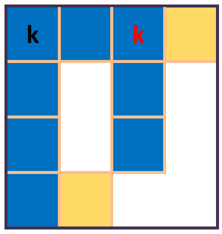

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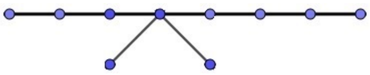

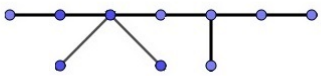

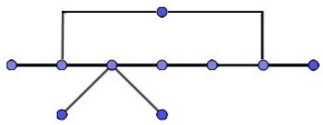

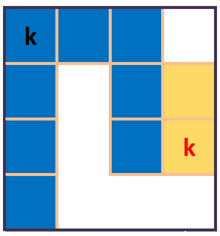

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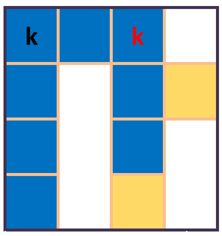

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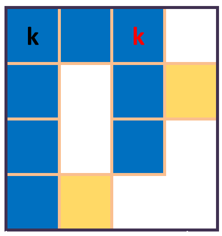

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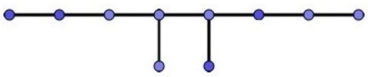

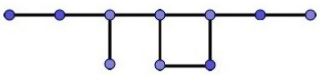

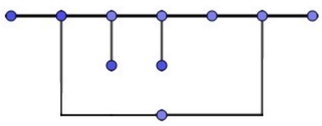

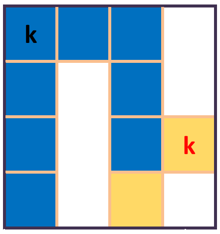

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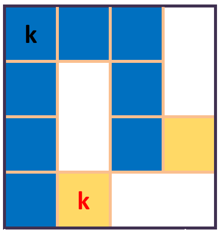

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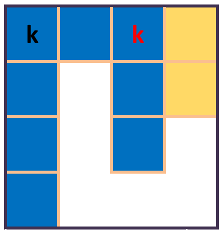

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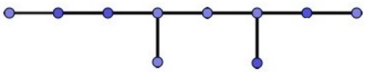

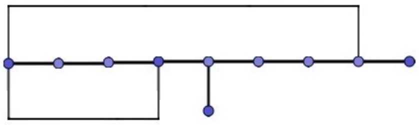

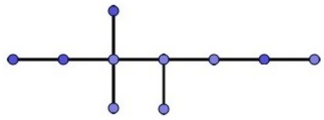

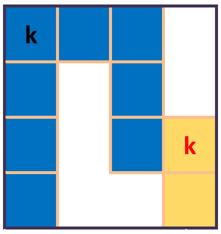

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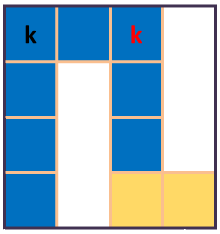

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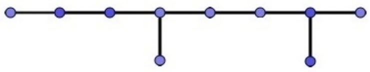
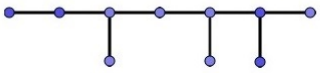
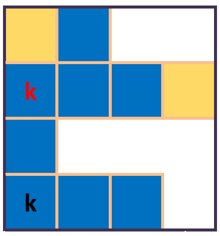




E 組　十連塊和節點圖



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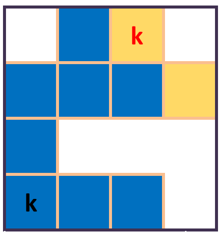

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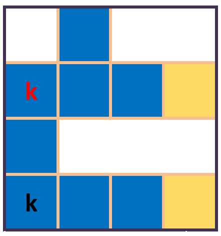

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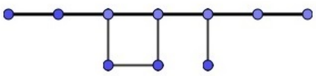

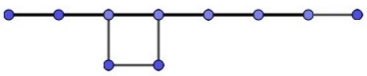

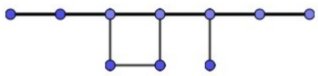

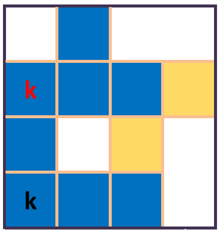

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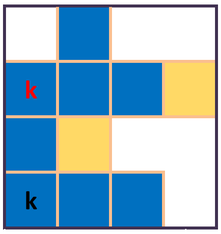

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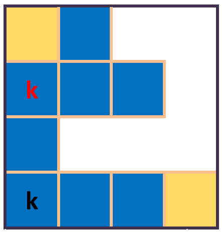

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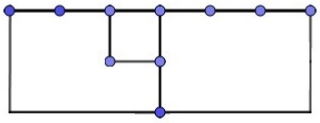

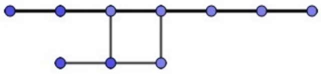

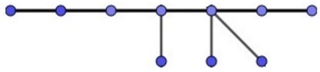

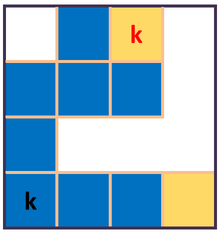

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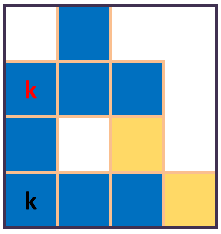

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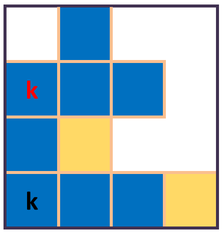

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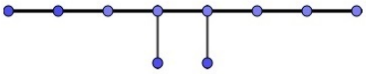

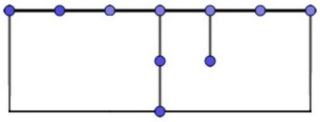

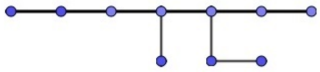

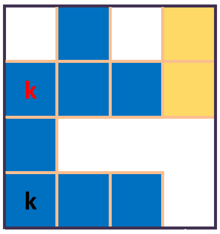

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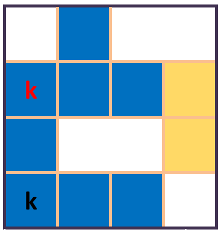

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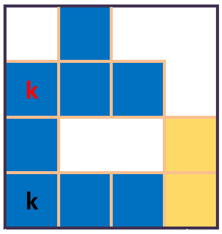

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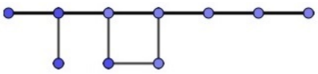

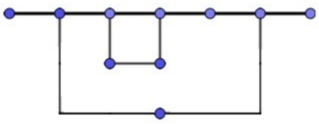

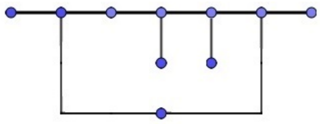

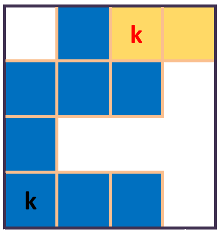

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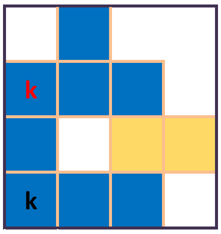

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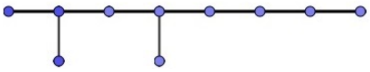
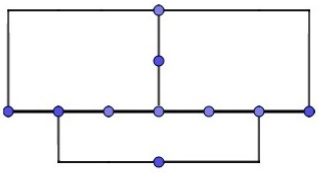
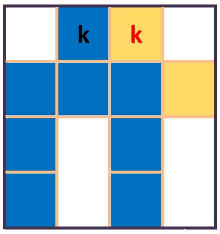




F 組 十連塊和節點圖



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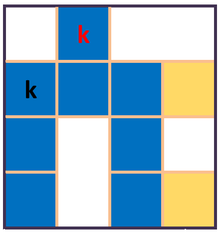

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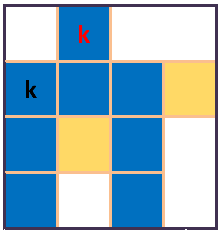

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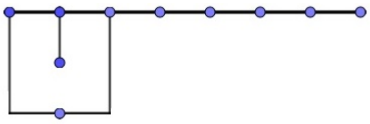

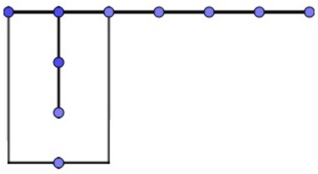

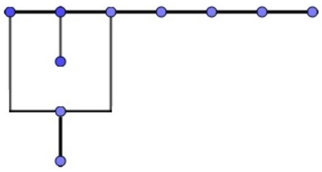

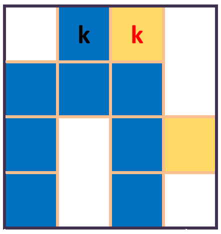

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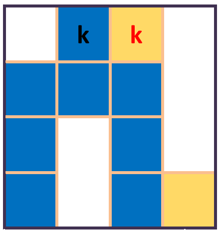

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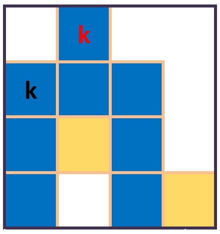

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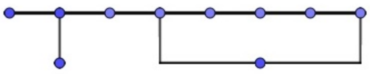

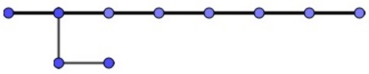

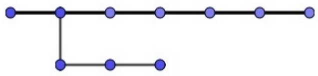

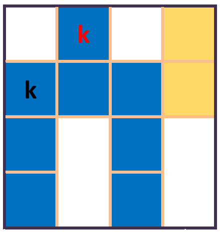

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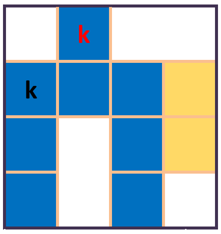

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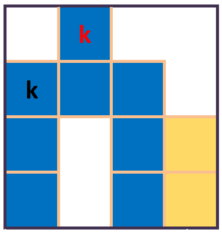

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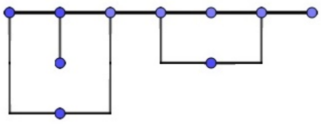

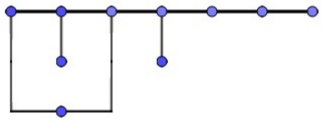

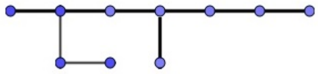

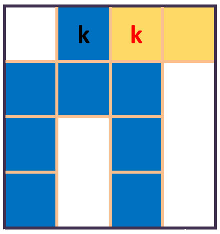

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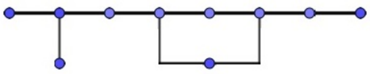
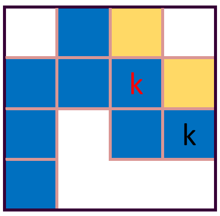


G 組 十連塊和節點圖



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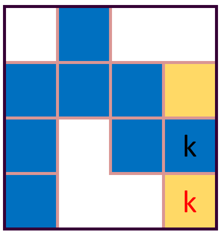

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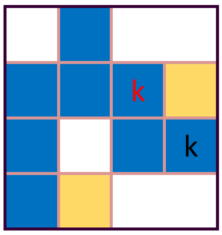

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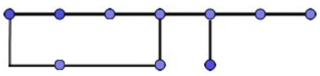

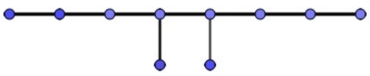

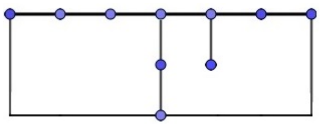

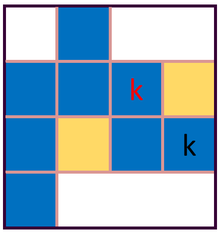

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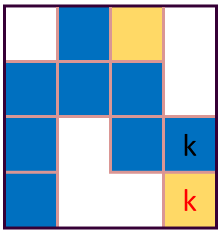

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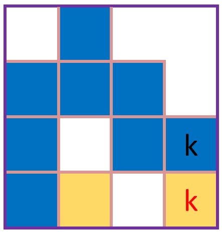

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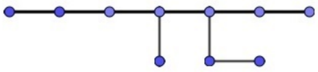

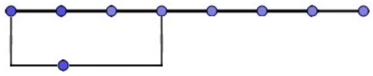

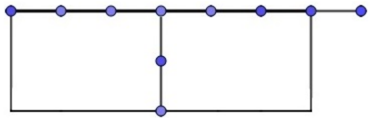

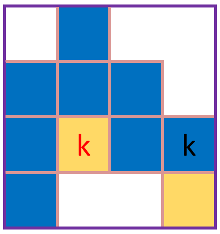

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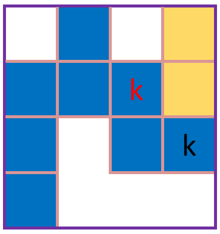

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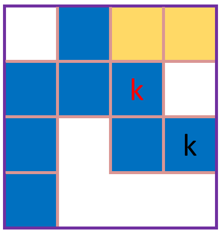

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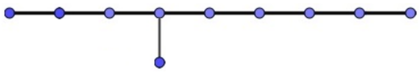
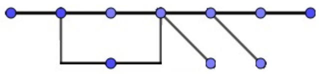
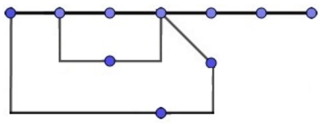






二、 節點與邊的數量

G={v , e}為節點與邊的組合，本次研究棋盤格節點都是 10 點，邊的數量則介於 9 到 12 之間。

A 組 節點與邊數量統計

編號	1	2	3	4	5	6	7	8	9	10	11
節點	10	10	10	10	10	10	10	10	10	10	10
邊	9	10	10	9	10	11	10	9	9	10	10

B 組 節點與邊數量統計

編號	1	2	3	4	5	6	7	8	9	10	11	12	13	14
節點	10	10	10	10	10	10	10	10	10	10	10	10	10	10
邊	10	9	9	9	9	10	9	9	10	10	10	9	11	9

C 組 節點與邊數量統計

編號	1	2	3	4	5	6	7
節點	10	10	10	10	10	10	10
邊	11	11	10	9	9	10	10

D 組 節點與邊數量統計

編號	1	2	3	4	5	6	7	8	9	10	11
節點	10	10	10	10	10	10	10	10	10	10	10
邊	9	9	10	9	10	10	9	11	9	9	9

E 組 節點與邊數量統計

編號	1	2	3	4	5	6	7	8	9	10	11	12	13	14
節點	10	10	10	10	10	10	10	10	10	10	10	10	10	10
邊	10	10	10	**12**	10	9	9	11	9	10	11	10	9	**12**

F 組 節點與邊數量統計

編號	1	2	3	4	5	6	7	8	9	10
節點	10	10	10	10	10	10	10	10	10	10
邊	10	10	10	10	9	9	11	10	9	10

G 組 節點與邊數量統計

編號	1	2	3	4	5	6	7	8	9
節點	10	10	10	10	10	10	10	10	10
邊	10	9	11	9	10	11	9	10	11

圖的尺寸介

圖的尺寸	A 組	B 組	C 組	D 組	E 組	F 組	G 組
9	4	8	2	7	4	3	3
10	6	5	3	3	6	6	3
11	1	1	2	1	2	1	3
12	0	0	0	0	2	0	0

新成形的十連塊在節點圖上有 10 個節點，76 個十連塊棋盤每一格騎士都會經過，沒有一個是虛格。換言之，回到八連塊 376 個來看，如果全部都加兩格到十連塊的話，有可能會出現更多虛格實格組合，可作為後續研究方向。

三、 可放置最多騎士並完成交換的可能性

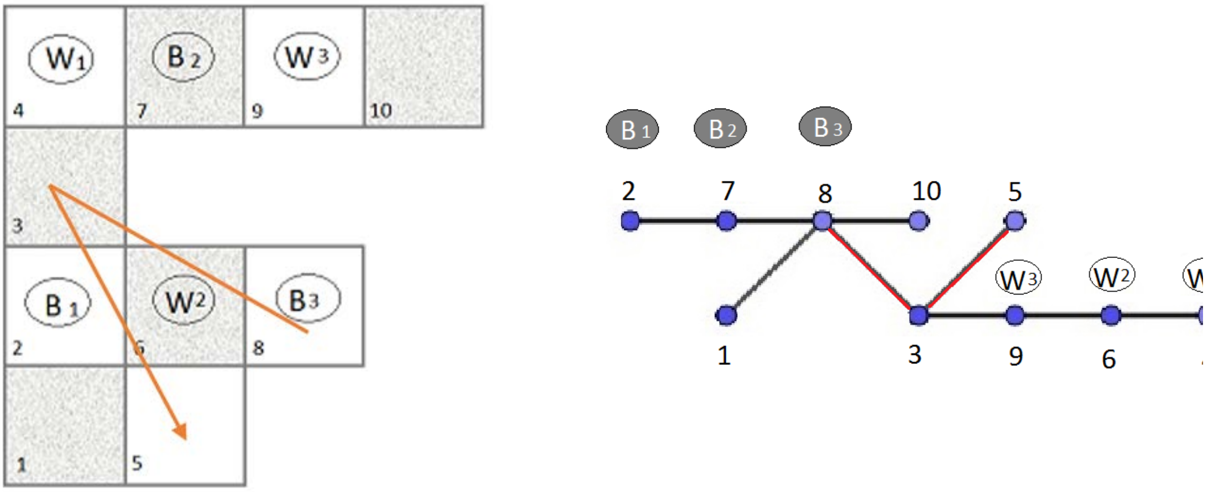
全部七組棋盤最少可以放置多少騎士呢?經由分解路徑後，我們得到至多可放置黑白各一子的棋盤有三個，分別是 **B-2**、**F -5**、**171-6**，可放置多達四黑四白的有 **D-8**。其他可放置兩黑兩白棋盤合計 35 個，三黑三白棋盤合計 37 個。

組別	1B1W	2B2W	3B3W	4B4W	合計
A		A-1 A-4 A-8 A-9 A-10	A-2 A-3 A-5 A-6 A-7 A-11	0	11
B	B-2	B-1 B-3 B-4 B-8 B-9 B-10	B-5 B-6 B-9 B-10 B-11 B-13	0	14
C		C-4 C-5	C-1 C-2 C-3 C-6 C-7	0	7
D		D-1 D-2 D-3 D-4 D-7 D-9 D-10 D-11	D-5 D-6	D-8	11
E		E-2 E-7 E-13	E-1 E-3 E-4 E-5 E-6 E-8 E-9 E-10 E-11 E-12 E-14	0	14
F	F-5 F-6	F-1 F-2 F-3 F-7 F-8 F-9 F-10	F-4	0	10
G		G-2 G-3 G-5 G-7	G-1 G-4 G-6 G-8 G-9	0	9
合計	3	35	37	1	76

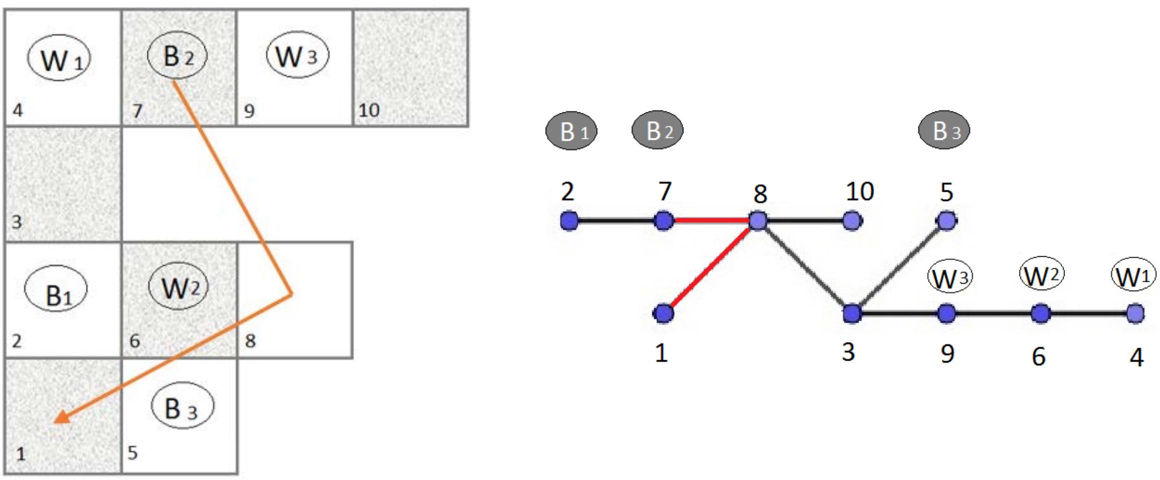
四、 騎士路徑與騎士交換

以 B-5 棋盤為例列出引導圖和棋盤上的相對應位置，由下圖可以看到三黑三白騎士，騎士移動路徑有 11 個步驟可達到騎士交換。

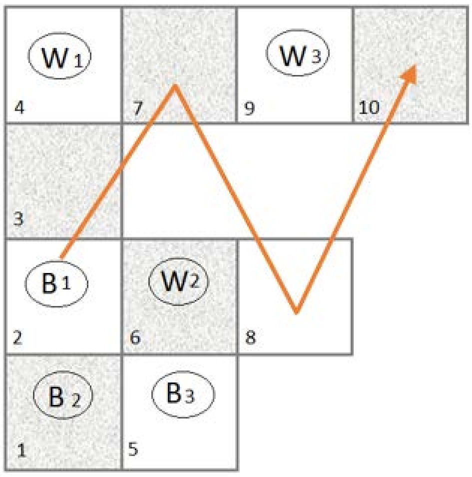
Step 1



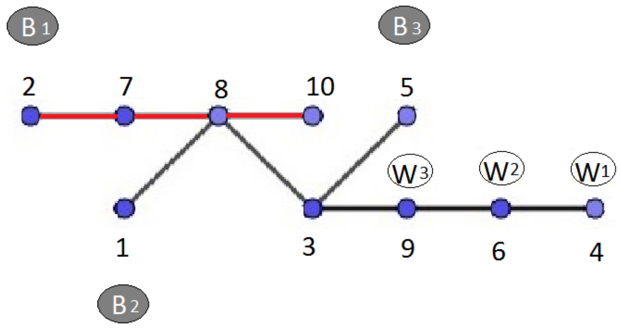
Step 2



Step 3

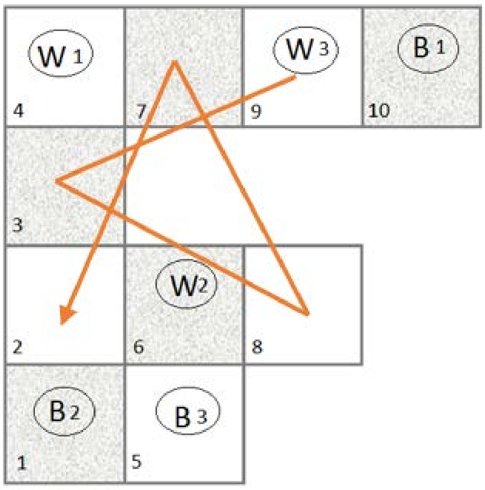


W1
W3
4
7
9
10
3
B1
W2
2
6
8
B2
B3
1
5

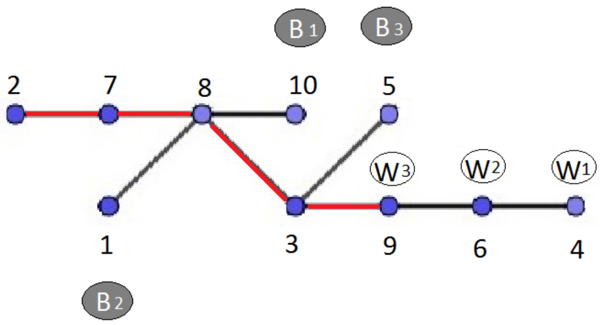


B1
B3
2
7
8
10
5
W3
W2
W1
1
3
9
6
4
B2

Step 4

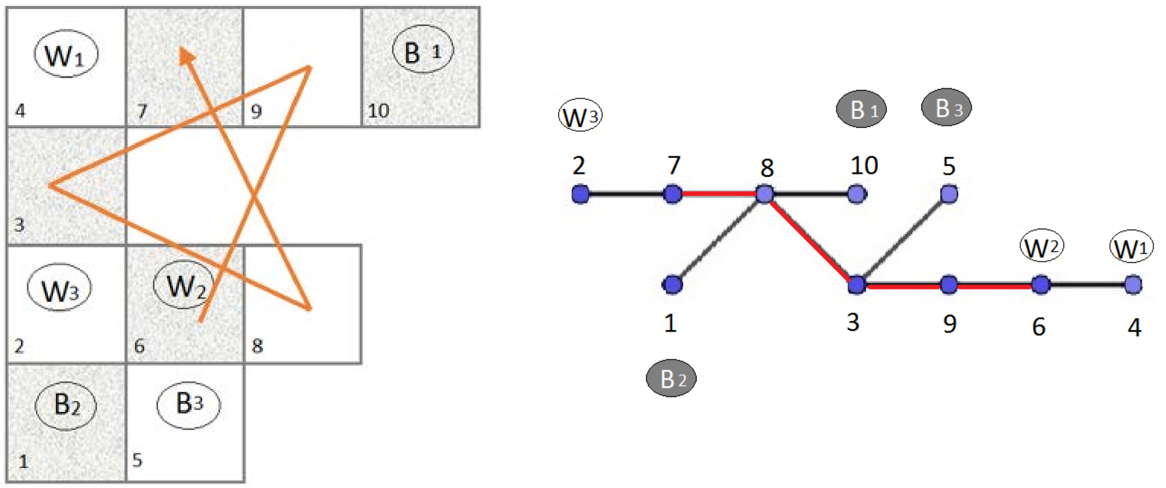


W1
W3
B1
4
7
9
10
3
W2
2
6
8
B2
B3
1
5



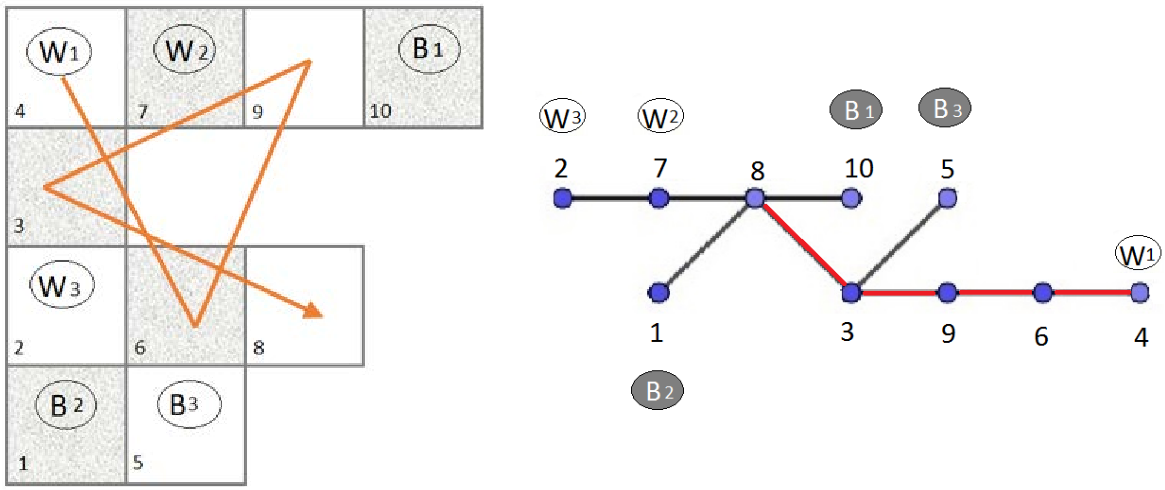
B1
B3
2
7
8
10
5
W3
W2
W1
1
3
9
6
4
B2

Step 5



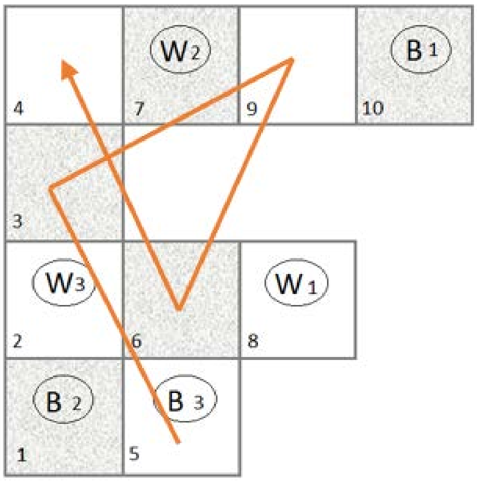
W1
4
7
9
B 1
10
3
W3
2
W2
6
8
B2
1
B3
5
W3
B 1
B 3
2
7
8
10
5
W2
W1
1
3
9
6
4
B 2

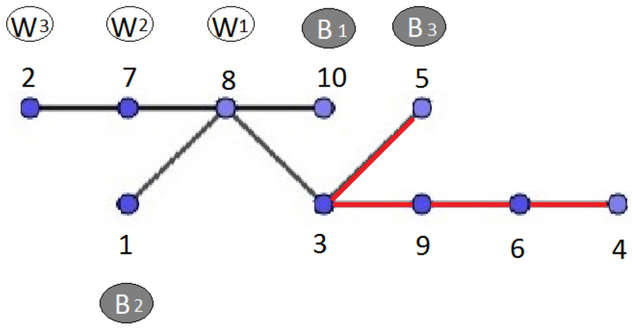
Step 6



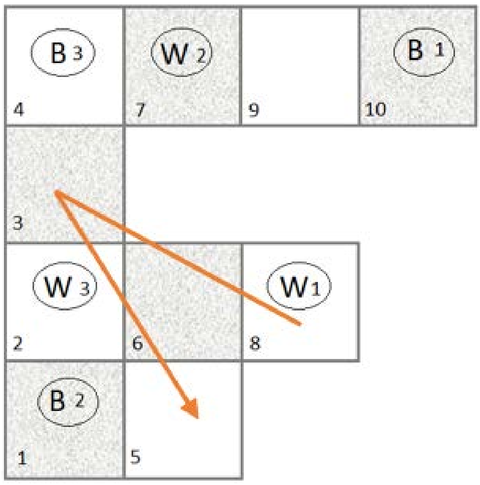
W1
4
W 2
7
9
B 1
10
3
W 3
2
6
8
B 2
1
B3
5
W3
W2
B 1
B 3
2
7
8
10
5
W1
1
3
9
6
4
B 2

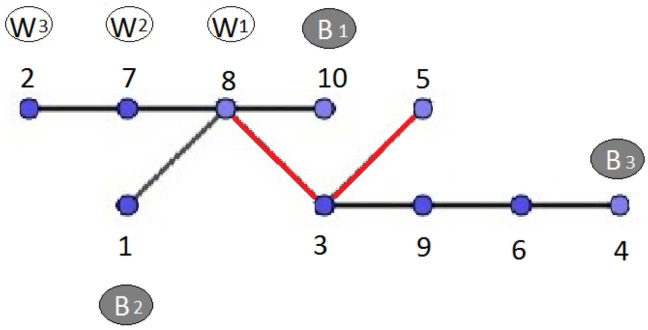
Step 7


W2
B1
4
7
9
10
3
W3
W1
2
6
8
B2
B3
1
5

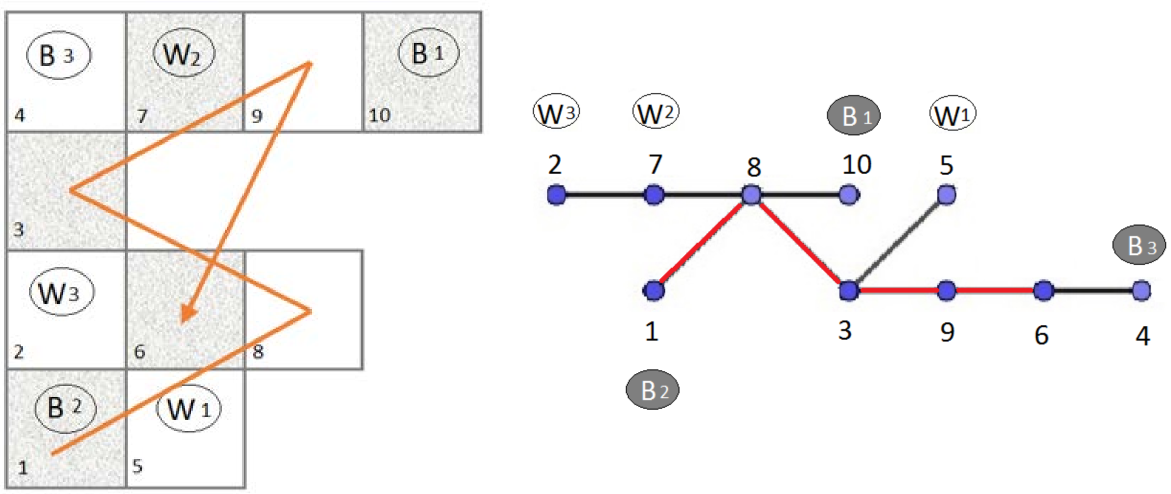

W3
W2
W1
B1
B3
2
7
8
10
5
1
3
9
6
4
B2

Step 8

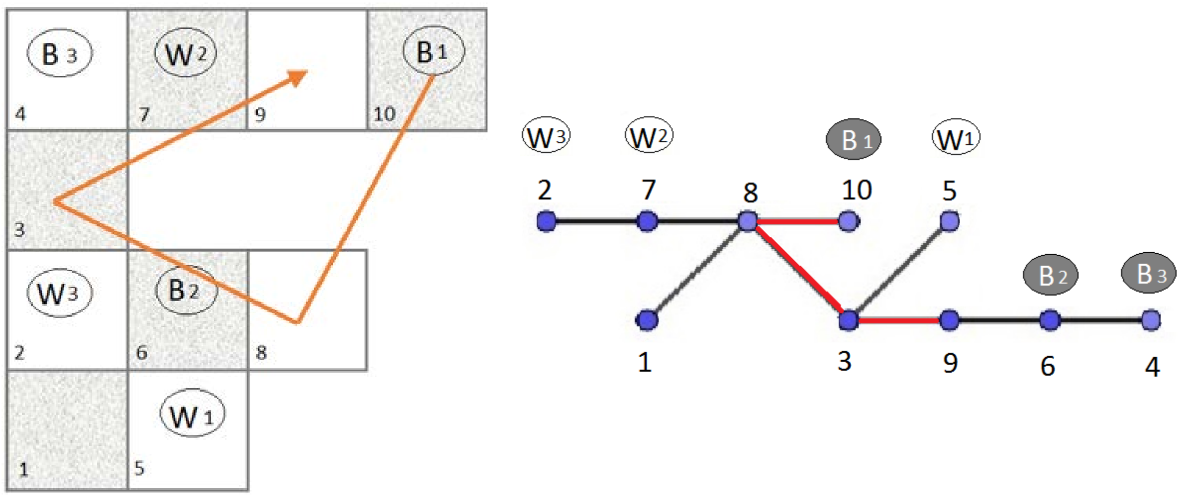

B3
W2
B1
4
7
9
10
3
W3
W1
2
6
8
B2
1
5


W3
W2
W1
B1
2
7
8
10
5
B3
1
3
9
6
4
B2

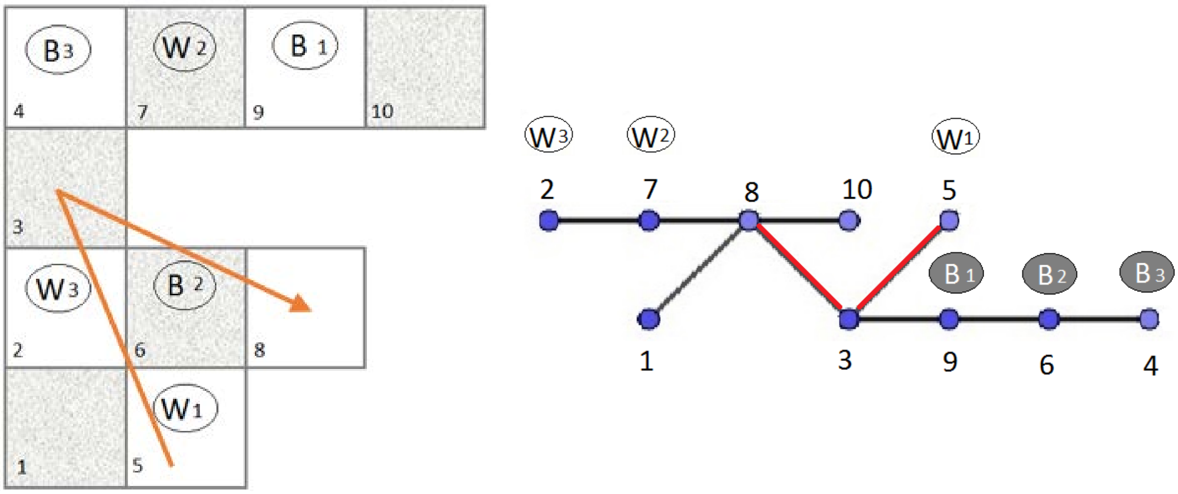
Step 9



Step 10



Step 11



四、　　騎士移動最少步數

進一步分解移動次數，依騎士數量及棋盤格型態和擺放位置有不同結果，一黑一白騎士得到最少移動步數是 3，兩黑兩白騎士最少移動步數是 6 步，三黑三白騎士最少移動步數是 9 步，四黑四白最少移動步數是 17 步。

陸、　研究結論

一、　以八連塊重組後得到 76 個 Crazy Knights 棋盤

我們從八連塊得到 7 個騎士巡邏圖，並以此衍生得到七組十連塊，合計 76 個瘋狂騎士棋盤格，可完成至少一組黑白騎士交換。

二、　節點與邊的數量

本次研究分析之節點圖皆有 10 點，圖的尺寸則介於 9 到 12 之間。

三、　可放置最多騎士並完成交換的可能性

全部七組棋盤至多可放置黑白各一子的棋盤有三個，可放置多達四黑四白的有一個，也是騎士對的極限。可放置兩黑兩白棋盤合計 35 個，三黑三白棋盤合計 37 個。

四、　騎士移動最少步數

移動次數依騎士數量及棋盤格型態和擺放位置有不同結果，一黑一白騎士得到最少移動步數是 3，兩黑兩白騎士最少移動步數是 6 步，三黑三白騎士最少移動步數是 9 步，四黑四白最少移動步數是 17 步。

參考文獻

張鎮華(2017)。演算法觀點的圖論。臺北市：台大出版中心出版。

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